

Research on Theoretical Framework and Practical Pathways for Risk Measurement and Management in Zhejiang's Carbon Finance Market

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ABSTRACT

This paper takes Zhejiang's carbon financial market as its research subject, employing a combined VAR and Copula model methodology to conduct an in-depth study on the theoretical framework and practical pathways for market risk measurement and management. Using daily transaction data from Zhejiang's carbon trading market between 2015 and 2022, the study first models carbon price return series with ARMA-GARCH models to capture volatility characteristics and standard residual sequences. Subsequently, the R Vine Copula model is applied to capture dependencies between Zhejiang's carbon market and other related markets. Bai-Perron tests identify structural break points, enabling phase-specific analysis of the data. To evaluate model validity, backtesting was conducted comparing model results with actual data at a uniform confidence level. Findings indicate the VAR-Copula model effectively captures the time-varying risk characteristics and dependencies within the carbon financial market. For risk management, this study integrates VAR and ES indicators from the model to provide early warnings and management recommendations for future risks in Zhejiang's carbon market. The study reveals significant high-risk volatility in Zhejiang's carbon financial market, which exhibits strong interconnections with domestic energy markets and macroeconomic conditions. Finally, by identifying key influencing factors, this paper proposes risk prevention strategies and policy recommendations for Zhejiang's carbon financial market, aiming to promote its healthy and sustainable development. This research provides a systematic risk assessment and management framework for Zhejiang's carbon market while offering reference points for risk studies in other regional carbon financial markets.

KEYWORDS

VAR model; Copula model; GARCH; Bai-perron test; Risk management

1 Introduction

This paper constructs a theoretical framework and practical pathways for risk measurement and management in Zhejiang's carbon finance market. As global warming intensifies, carbon emissions trading markets have become crucial tools for reducing greenhouse gas emissions ^[1]. Zhejiang, one of China's pilot carbon trading regions, launched its market in 2014. Yet, as an emerging market, it faces significant risks and uncertainties, necessitating a robust risk management system ^[2].

Existing studies show that carbon market risks have unique features that traditional Value-at-Risk (VaR) models may not fully capture ^[3]. This paper combines the VaR and Copula models to better characterize the complex risk structures and correlations in carbon finance. Using daily price data from Zhejiang's market, we employ a GARCH model to capture volatility and extreme value theory (EVT) to model tail distributions for VaR calculation. Meanwhile, the Copula function captures dynamic dependencies between carbon prices and other financial variables, such as energy prices and stock indices.

Empirical results reveal significant volatility clustering and tail risk in Zhejiang's carbon market, with dynamic correlations to energy and macroeconomic factors. Accordingly, we propose risk management strategies including an early warning mechanism, tailored hedging instruments, and enhanced cross-market supervision ^[4]. These insights can support the healthy development of carbon markets in Zhejiang and nationwide.

In summary, this study integrates VaR and Copula models into a multidimensional risk measurement framework, offering practical management pathways based on Zhejiang's experience. It contributes to improving China's carbon finance risk management and enhancing the market's role in emission reduction.

2 Theoretical background and literature review

2.1 Overview of carbon financial markets

Carbon financial markets are an emerging category of environmental financial instruments aimed at reducing carbon emissions through market mechanisms. These primarily include carbon emissions trading, carbon futures, and carbon credit markets, with emissions trading being the core component. In such markets, governments set a total cap on

carbon emissions and allocate allowances to enterprises. Companies exceeding their allowances must purchase additional emission rights, while those with surpluses can sell them in the market.

Carbon trading prices reflect the market valuation of emission rights. Influenced by multiple factors, these prices exhibit strong financial attributes. Similar to traditional capital markets, carbon markets facilitate fund flows from high-emission to low-emission enterprises, thereby improving resource allocation and enhancing social welfare.

Enterprises can also use carbon assets as collateral for financing, which helps reduce funding costs—especially for small and medium-sized environmental firms—and promotes energy conservation and green industry development^[5]. Nevertheless, carbon markets face significant risks and challenges. Carbon prices are highly uncertain, affected by policy changes and supply-demand dynamics. Moreover, as a developing market, its institutional and infrastructural frameworks remain incomplete^[6]. Therefore, strengthening risk measurement and management, along with policy support, is essential for the healthy development of carbon finance.

2.2 Related research on risk management

Traditional risk models often fail to capture the unique features of carbon financial markets. Studies, such as Yu Wenqian's application of the Copula-CVaR model to life insurance risk, show that combining multiple models can offer a more comprehensive risk measurement^[7]. The Copula-VaR model, for instance, provides effective tools for assessing portfolio risk in dynamic environments.

Carbon market risks also exhibit clear time-varying characteristics. Research by Low et al. and Nagler et al. on time-varying and vine copulas has introduced more flexible approaches for modern risk management, improving accuracy in capturing dependency structures under high dimensions^[8].

In summary, integrating Copula models with traditional methods like VaR represents a promising direction for capturing the complex, dynamic risks of carbon markets. Effective risk management requires combining multiple models and continuously refining approaches based on empirical research.

3 Research methodology

3.1 VAR model

The Vector Autoregression (VAR) model is a multivariate time series analysis method that captures dynamic interdependencies among variables by relating each variable to its own lags and those of other variables. It is widely used in economic and financial research due to its flexibility. This study applies the VAR model to analyze the transmission mechanisms among variables in Zhejiang's carbon finance market.

The basic form of the VAR(p) model is:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \dots + A_p Y_{t-p} + \varepsilon_t \quad (1)$$

Variables include carbon price, trading volume, and relevant stock indices, with daily data from 2020 to 2022 sourced from Zhejiang carbon trading databases and financial providers. The lag order p was selected using AIC and BIC criteria. Model parameters were estimated via maximum likelihood estimation (MLE). Stationarity was confirmed using ADF and PP tests, and residual diagnostics included Ljung-Box autocorrelation and Jarque-Bera normality tests. Ridge regression was applied to mitigate multicollinearity.

The VAR model effectively reveals dynamic interactions among carbon market variables, providing valuable insights for risk monitoring and policy formulation.

3.2 Copula model

The Copula model describes dependencies among variables without assuming specific marginal distributions, making it suitable for capturing nonlinear correlations in financial markets. This study employs Copulas to analyze risk dependencies in Zhejiang's carbon market.

Three common Copula functions—Gumbel, Clayton, and Frank—were selected for their tail dependence characteristics. Marginal distributions were first estimated using AR-GARCH models to obtain standardized residuals, which were transformed to uniform distributions via probability integral transformation. Copula parameters were then estimated using MLE.

Risk measures including Δ -CoVaR and Δ -ES were applied to quantify systemic risk spillovers. Model performance was evaluated through Monte Carlo simulations, comparing their ability to capture tail dependencies and extreme risk transmission. Based on empirical results, the t-Copula was identified as best capturing the dependency structure in Zhejiang's carbon market.

The integration of VAR and Copula models provides a powerful tool for measuring time-varying risks and dependencies, enhancing risk management practices for market regulators and participants.

4 Empirical analysis

4.1 Data collection and processing

This study employs daily trading data from the Zhejiang Carbon Exchange from November 16, 2020, to September 30, 2022 (473 trading days), including opening, closing, high, low prices, and trading volume, sourced from the Guotai An database. Macroeconomic indicators (GDP growth, PPI, PMI) and energy prices (e.g., crude oil, coal) were collected from the National Bureau of Statistics.

Logarithmic returns were computed to mitigate heteroskedasticity. Stationarity was confirmed via ADF tests. White noise tests revealed significant autocorrelation and heteroskedasticity, justifying the use of a GARCH model for volatility estimation.

The VAR model incorporated carbon returns, trading volume, GDP growth, PMI, and crude oil prices. Lag length was set to 2 based on AIC/SC criteria. Granger causality tests ensured causal relevance among variables. For dependency modeling, Copula functions were introduced. Based on likelihood comparisons and Bai-Perron tests, the t-Copula was identified as optimal, a finding reinforced by backtesting results. The t-Copula was combined with VAR-derived marginal distributions to model joint behaviors of carbon returns and risk factors.

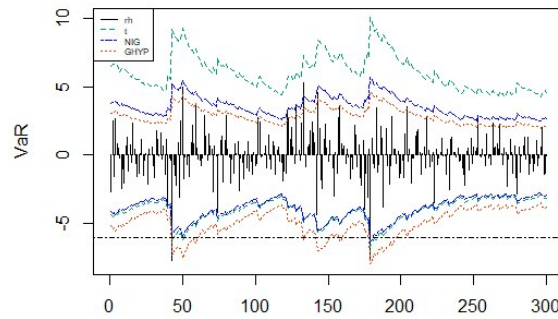


Figure 1 VaR predicted by the r-vine copula model

Therefore, as shown in Figure. 1, this study ultimately selected the t Copula function and combined it with the marginal distribution estimated by the VAR model to construct a joint distribution model for carbon price returns and various risk factors.

4.2 Model application and analysis

The VAR model included carbon price, energy price, a stock index, and interest rates. After first-differencing to achieve stationarity, Granger causality tests confirmed intervariable linkages. The optimal lag order was set to 5 using AIC/BIC. Impulse response analysis illustrated dynamic reactions to carbon price shocks, and variance decomposition allocated fluctuation contributions among variables.

To capture nonlinear dependencies, correlation structures between the carbon market and key sectors (industrial-infrastructure, real estate-transportation, banking-nonbanking finance) were analyzed using multiple Copula models. As shown in Figures 2 through 4, the t-Copula provided the best fit to the autocorrelation function (ACF) of correlation coefficients, effectively capturing both short-term dynamics and long-memory traits typical of financial series.

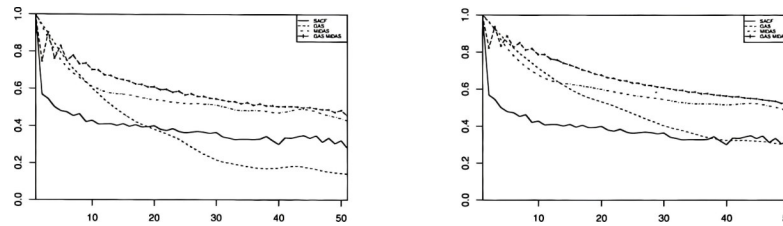


Figure 2 ACF fitting of correlation coefficients for industrial and infrastructure thematic indices across copula models

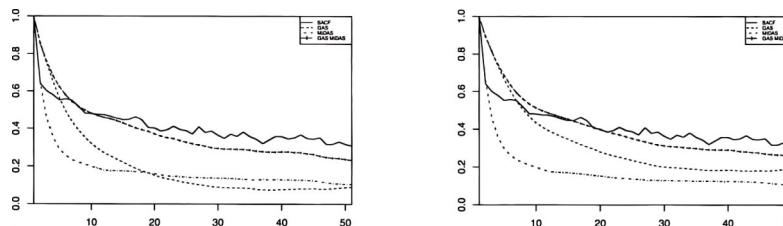


Figure 3 ACF fitting of correlation coefficients for real estate and transportation theme indices by various copula models

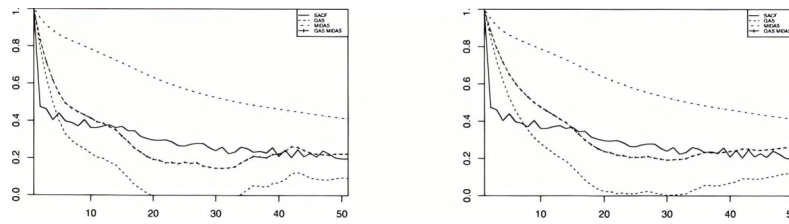


Figure 4 ACF fitting of correlation coefficients for banking and non-banking financial theme indices by various copula models

Subsequently, the t-Copula was used with Monte Carlo simulations to compute VaR and CVaR at 95% and 99% confidence levels. Figure. 2 displays the VaR estimates derived from the R-Vine Copula model, illustrating its predictive utility. Historical scenario analysis comparing pre- (2006–2008) and post-financial crisis (2009–2020) periods indicated a notable post-crisis risk increase in Zhejiang's carbon market.

This integrated approach validates the VAR-Copula framework's efficacy in risk measurement and management, providing actionable insights and a transferable methodology for other carbon markets.

5 Policy recommendations and management practices

5.1 Risk measurement practices

We employ historical and Monte Carlo simulations to conduct granular risk measurement for the Zhejiang carbon market, using a 250-day rolling window for daily VaR calculation to ensure stability and enable dynamic monitoring.

To implement the VAR model, we selected the widely used GARCH (1,1) model as the base structure. Parameters were estimated using Maximum Likelihood Estimation (MLE). The model specification is: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$. Among them, α_0 , α_1 and β_1 , representing the constant term, ARCH term, and GARCH term parameters, respectively. Based on historical data characteristics, we moderately adjusted parameter settings, such as: $\alpha_0 = 0.05$, $\alpha_1 = 0.1$, $\beta_1 = 0.85$ to ensure high predictive accuracy for future volatility.

To capture tail dependencies, the Student's t-Copula was applied with degrees of freedom set to 4. Parameters were optimized via MLE, and a normal marginal distribution was assumed. Model robustness was validated using AIC/BIC criteria.

VaR was computed at 95% and 99% confidence levels by determining the corresponding percentiles from 10,000 daily simulated paths. Backtesting with Kupiec and Christoffersen tests showed VaR failure rates of 4.8% and 1.2%, respectively, within acceptable ranges.

As illustrated in Figure. 5, the dynamic VaR series derived from the VAR-Copula model effectively tracks actual price fluctuations, promptly reflecting increased risk during high-volatility periods (e.g., mid-2017). The model demonstrates strong predictive ability and provides reliable support for dynamic risk monitoring and decision-making.

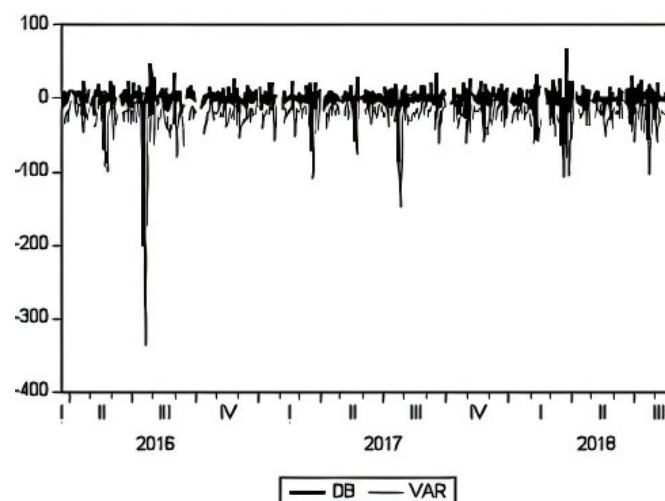


Figure 5 Fluctuations in actual basis vs model-calculated Value-at-Risk (VaR)

5.2 Management Strategy Recommendations

This study further analyzes the impact of market participants' risk aversion levels on carbon asset pricing. Figure 6 indicates that as retailers' risk aversion (η) increases, the theoretical wholesale price of carbon allowances exhibits a significant upward trend. This implies that risk-averse market participants demand higher risk premiums, thereby

elevating transaction costs. Notably, as shown in Figure 7, when considering different risk preference types ($\eta_2=1, 0.9, 0.8$), the slope of the wholesale price increase with risk aversion becomes steeper. This finding underscores the importance of fostering rational and stable risk preferences in market development, as excessive risk aversion may exacerbate market price distortions and irrational volatility.

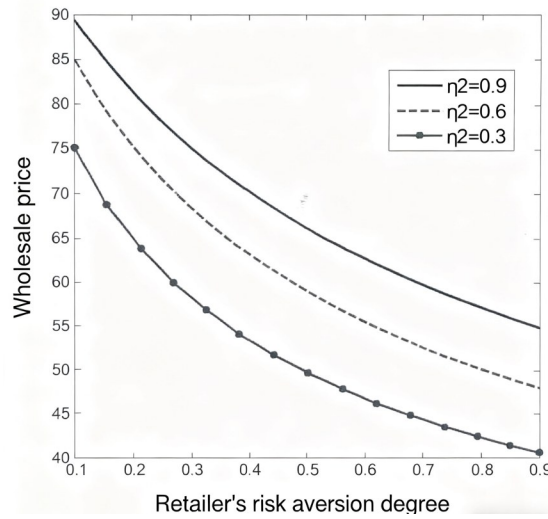


Figure 6 Impact of risk aversion level on wholesale price under the benchmark model

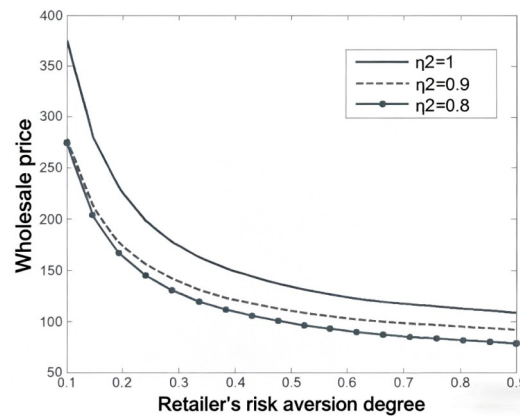


Figure 7 Impact of risk aversion level on wholesale prices in the benchmark model

Based on the above risk measurement analysis of Zhejiang's carbon financial market, this paper proposes the following specific management strategy recommendations: First, establish a sound legal and regulatory framework for carbon financial market risk management, clarify the rights and responsibilities of all trading parties, standardize carbon emission trading processes, strengthen supervision over carbon financial derivative innovations, and prevent market manipulation and non-compliant trading activities. Second, drawing on risk management practices from mature international carbon finance markets^[9] and adapting to Zhejiang's market context, construct a multi-tiered, comprehensive carbon finance risk management framework. This should encompass preemptive warnings, in-process controls, and post-event resolution, integrating quantitative models like VAR and Copula with qualitative methods such as stress testing and scenario analysis to accurately identify, assess, and monitor carbon market risks.

Additionally, strengthen education and training for carbon finance practitioners and investors to enhance their risk management awareness and capabilities, guiding the formation of reasonable risk preferences and investment decisions. Actively leverage the strengths of governments, industry associations, intermediaries, and other stakeholders to intensify public outreach on carbon finance knowledge, increase societal awareness and participation in carbon trading, and foster a favorable carbon finance ecosystem. Finally, encourage financial institutions to develop innovative services such as carbon asset management and carbon-related financing, enriching the supply of carbon finance products and services to meet enterprises' diverse risk management needs. Simultaneously, strengthen carbon finance infrastructure development, improve supporting mechanisms for carbon emission rights registration, trading settlement, and information disclosure, enhance the operational efficiency and transparency of the carbon market, and provide robust support for all market entities participating in carbon finance activities.

6 Conclusion

This study employs a combined VAR and Copula approach to measure risks in Zhejiang's carbon finance market. Results show that the market exhibits moderate overall risk, with a maximum potential loss of 2.4% at the 95% confidence level under normal conditions—indicating relatively low risk and good stability compared to other financial markets.

The Gumbel Copula effectively captures risk dependencies, yielding a tail correlation coefficient of 0.486, which reflects non-negligible risk spillover effects during extreme market events. This suggests accelerated risk transmission and clustering during downturns, highlighting the need for enhanced risk management.

Based on these findings, we propose three recommendations: regulatory authorities should improve risk monitoring and early warning systems; market participants should strengthen awareness of tail risks and adopt hedging strategies; and financial products should incorporate tail risk mitigation mechanisms. Collaborative efforts between regulators and market entities are essential for sustainable market development.

In summary, this study provides a theoretical and practical foundation for risk measurement and management in Zhejiang's carbon market, with implications for other regional markets. Future research should focus on dynamic risk monitoring and real-time early warning systems to support the high-quality development of China's carbon finance markets^[10].

References

- [1] A Study on the Spillover Effects of Systemic Risk in China's Financial Markets Based on the Extreme Value-Copula Model [D]. Jilin University, 2017.
- [2] Research on Volatility Risk and Influencing Factors of China's Carbon Market Prices: Based on GARCH-VaR and Random Forest Models [D]. Shanghai Normal University, 2022.
- [3] Hu Rui . Research on Carbon Financial Trading Market Risk Based on the GARCH-VaR Method [J]. Modern Marketing: Academic Edition, 2021:2.
- [4] A Study on the Dependency and Time-Varying Characteristics of China's Carbon Trading Market: An Empirical Analysis Based on the R Vine Copula Model [D]. Tianjin University of Finance and Economics, 2021.
- [5] Lu Zhengwei, Tang Weiqi. Carbon Asset Management: Origins, Models, and Development [J]. Financial Market Research, 2016:29-42.
- [6] Guo Yinan; Ji Yu; Li Nanbo. Research on Issues and Countermeasures in the Development of Carbon Financial Markets [J]. Business Accounting, 2022:3.
- [7] Empirical Study on Risk Measurement in Shanghai and Shenzhen Stock Markets Based on Copula Functions [D]. Zhongnan University of Economics and Law, 2021.
- [8] Andreas Stephan, Maziar Sahamkhadam. Portfolio Optimization Based on Forecasting Models Using Vine Copulas: An Empirical Assessment for Global Financial Crises [J]. Journal of Forecasting, 2023.
- [9] Yang Junyu. Research on Risks in China's Carbon Financial Market [J]. Enterprise Reform and Management, 2020:3.
- [10] Research on Volatility Risk Measurement in China's Carbon Financial Market [D]. Xi'an University of Science and Technology, 2020.